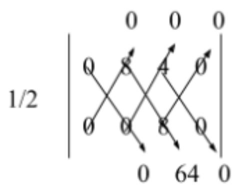
Rocket City Math League — 2018-2019
Apollo Solutions — Round 1

1. **Answer: $\mathbf{52 - 4i}$**

   $(8+4i)(5-3i) = 40 - 24i + 20i - 12i^2$. Since $i^2 = -1$, this simplifies to $40 - 24i + 20i + 12 = 52 - 4i$.

2. **Answer: 32**

   Using the shoelace method:

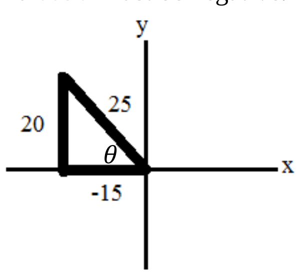


   Thus, the area equals $\frac{1}{2}[(0+64+0)-(0+0+0)] = 32$

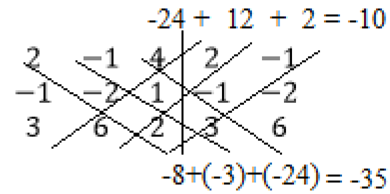
   Alternatively, using the distance formula:
   $\sqrt{(0-0)^2+(0-8)^2} = 8$
   $\sqrt{(0-8)^2+(8-4)^2} = \sqrt{80} = 4\sqrt{5}$
   $\sqrt{(0-8)^2+(0-4)^2} = \sqrt{80} = 4\sqrt{5}$
   These three side lengths form an isosceles triangle. The Pythagorean Theorem gives 8 as h. Using the $\frac{1}{2}bh$ formula for area and the 8 side as b, $\frac{1}{2}(8)(8) = 32$.

3. **Answer: $\mathbf{\frac{4}{5}}$**

   If $\cot\theta = -\frac{15}{20}$, then $\tan\theta = -\frac{20}{15}$. Since $90° < \theta < 180°$, $\sin\theta$ must be positive and $\cos\theta$ must be negative. So, $\sin\theta = \frac{20}{25} = \frac{4}{5}$.



4. **Answer: $\mathbf{-25}$**



   Therefore, the determinant is $-35 - (-10) = -25$.

5. **Answer: $\mathbf{81\pi}$**

   After completing the square, the space that Hector can roam, $(x-4)^2 + (y-3)^2 = 25$, has a radius of 5 feet. If Sam adds 4 feet to this, Hector's leash is now 9 feet long, which means his play area is $\mathbf{81\pi}$.

6. **Answer: Day 8**

   If the pattern continues, then Sally sells 9 more vanilla ice creams and 11 more chocolate ice creams each day than the day before. Thus, the number of vanilla and chocolate ice creams sold is as follows:

Day	1	2	3	4	5	6	7	8
Vanilla	21	30	39	48	57	66	75	84
Chocolate	7	18	29	40	51	62	73	84

   Thus, Sally sells the same number of vanilla and chocolate ice creams on Day 8. Alternatively, let n be the number of days that has passed after the first day of business. Then, $21 + 9n = 7 + 11n$. Solving for n gives $n = 7$, so 7 days after the first day of business, or on the 8th day, Sally sells the same number of vanilla and chocolate ice creams.

7. **Answer: $\mathbf{\frac{7}{4}}$**

   Since $\sin^2\theta + \cos^2\theta = 1$, $\sin^2\frac{13\pi}{3} + \cos^2\frac{13\pi}{3} + \sin^2\frac{14\pi}{3} = 1 + \sin^2\frac{14\pi}{3}$. Also, $\frac{14\pi}{3} = \frac{2\pi}{3}$, so $1 + \sin^2\frac{14\pi}{3} = 1 + \sin^2\frac{2\pi}{3} = 1 + \left(\frac{\sqrt{3}}{2}\right)^2 = 1 + \frac{3}{4} = \frac{7}{4}$.

8. **Answer: -191**

Use synthetic division:

-2	7	0	-5	0	3	-1
		-14	28	-46	92	-190
	7	-14	23	-46	95	$\boxed{-191}$

9. **Answer: 85**

Let x equal the number of Mars Macaroons, y equal the number of Mini Pavlovas, and z equal the number of Nova Bites that Agatha buys. Then, $x + y + z = 68$, $3x + 4y + 5z = 260$, and $z = \frac{1}{3}x + \frac{1}{3}y$.
Substituting the third equation into the first equation for z gives $\frac{4}{3}x + \frac{4}{3}y = 68$, and substituting the third equation into the second equation gives $\frac{14}{3}x + \frac{17}{3}y = 260$.
Now, multiplying the new first equation by 7 and the new second equation by 2 and subtracting the first from the second gives $2y = 44$, so $y = 22$. Substituting back into one of the equations gives $x = 29$. Substituting into the original third equation gives $z = \frac{1}{3}(29) + \frac{1}{3}(22) = 17$. Agatha bought 5 Nova Bites, so she paid $17 \cdot 5 = 85$ coins.

10. **Answer: 15**

Simplify using logarithm rules:

$\log_{1/3}(\log_{64}(\log_2(x+1))) = \log_{25} 10 - \log_{25} 2 + \log_{49} 7$
$\log_{1/3}(\log_{64}(\log_2(x+1))) = \log_{25} 5 + \log_{49} 7$
$\log_{1/3}(\log_{64}(\log_2(x+1))) = \dfrac{1}{2} + \dfrac{1}{2} = 1$
$\log_{64}(\log_2(x+1)) = \dfrac{1}{3}$
$\log_2(x+1) = 4$
$x + 1 = 16$
$x = 15.$