NAME _____ DATE _____ PERIOD _____

7-2

Study Guide

Verifying Trigonometric Identities

When verifying trigonometric identities, you cannot add or subtract quantities from each side of the identity. An unverified identity is not an equation, so the properties of equality do not apply.

Example 1 **Verify that $\frac{\sec^2 x - 1}{\sec^2 x} = \sin^2 x$ is an identity.**

Since the left side is more complicated, transform it into the expression on the right.

$$\frac{\sec^2 x - 1}{\sec^2 x} \stackrel{?}{=} \sin^2 x$$

$$\frac{(\tan^2 x + 1) - 1}{\sec^2 x} \stackrel{?}{=} \sin^2 x \qquad \sec^2 x = \tan^2 x + 1$$

$$\frac{\tan^2 x}{\sec^2 x} \stackrel{?}{=} \sin^2 x \qquad \textit{Simplify.}$$

$$\frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}} \stackrel{?}{=} \sin^2 x \qquad \tan^2 x = \frac{\sin^2 x}{\cos^2 x},\ \sec^2 x = \frac{1}{\cos^2 x}$$

$$\frac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x \stackrel{?}{=} \sin^2 x$$

$$\sin^2 x = \sin^2 x \qquad \textit{Multiply.}$$

The techniques that you use to verify trigonometric identities can also be used to simplify trigonometric equations.

Example 2 **Find a numerical value of one trigonometric function of x if $\cos x \csc x = 3$.**

You can simplify the trigonometric epression on the left side by writing it in terms of sine and cosine.

$$\cos x \csc x = 3$$

$$\cos x \cdot \frac{1}{\sin x} = 3 \qquad \csc x = \frac{1}{\sin x}$$

$$\frac{\cos x}{\sin x} = 3 \qquad \textit{Multiply.}$$

$$\cot x = 3 \qquad \cot x = \frac{\cos x}{\sin x}$$

Therefore, if $\cos x \csc x = 3$, then $\cot x = 3$.

NAME _____ DATE _____ PERIOD _____

7-2

Practice

Verifying Trigonometric Identities

Verify that each equation is an identity.

1. $\frac{\csc x}{\cot x + \tan x} = \cos x$

2. $\frac{1}{\sin y - 1} - \frac{1}{\sin y + 1} = -2 \sec^2 y$

3. $\sin^3 x - \cos^3 x = (1 + \sin x \cos x)(\sin x - \cos x)$

4. $\tan \theta + \frac{\cos \theta}{1 + \sin \theta} = \sec \theta$

Find a numerical value of one trigonometric function of x.

5. $\sin x \cot x = 1$

6. $\sin x = 3 \cos x$

7. $\cos x = \cot x$

8. ***Physics*** The work done in moving an object is given by the formula $W = Fd \cos \theta$, where d is the displacement, F is the force exerted, and θ is the angle between the displacement and the force. Verify that $W = Fd \frac{\cot \theta}{\csc \theta}$ is an equivalent formula.

NAME _____ DATE _____ PERIOD _____

7-2

Enrichment

Building from 1 = 1

By starting with the most fundamental identity of all, 1 = 1, you can create new identities as complex as you would like them to be.

First, think of ways to write 1 using trigonometric identities. Some examples are the following.

$$1 = \cos A \sec A$$

$$1 = \csc^2 A - \cot^2 A$$

$$1 = \frac{\cos (A + 360°)}{\cos (360° - A)}$$

Choose two such expressions and write a new identity.

$$\cos A \sec A = \csc^2 A - \cot^2 A$$

Now multiply the terms of the identity by the terms of another identity of your choosing, preferably one that will allow some simplification upon multiplication.

$$\begin{array}{rl} \cos A \sec A = \csc^2 A - \cot^2 A \\ \times \quad \frac{\sin A}{\cos A} = \tan A \\ \hline \sin A \sec A = \tan A \csc^2 A - \cot A \end{array}$$

Beginning with 1 = 1, create two trigonometric identities.

1. _____

2. _____

Verify that each of the identities you created is an identity.

3. _____

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4. _____

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