2014 – 2015 Log1 Contest Round 1
Theta Applications

Name: _____

4 points each		
1	An experiment has a maximum theoretical yield of 25 moles. When Danae runs the experiment, she measures her yield as 22.5 moles. What was Danae's percent error from the maximum in running the experiment?	
2	If it takes Connor 5 hours to paint a house by himself, and if it take Zheye 8 hours to paint the same house by himself, how many hours would it take Connor and Zheye, working independently, to paint the house together?	
3	One train leaves Prague at 6 am, traveling at a rate of 40 miles/hour. Another train leaves Prague, on a track parallel to the first train's track, at 7 am, traveling at a rate of 50 miles/per hour. At what time will the two trains have traveled the same amount?	
4	How many mL of a 50% acid solution should be added to 10 mL of pure water to create a solution that is 30% acid?	
5	A father is currently eight times as old as his daughter, and his daughter is currently twice as old as his son. If the sum of their three current ages is 38, how old will the son be ten years from now?	

5 points each		
6	Ishant drops a ball from the top of a 250-foot tall building. The ball falls straight down, and each time it bounces off the ground, it rebounds to $\frac{1}{6}$ of the height from which it fell. What is the total distance traveled by the ball, in feet, before it comes to rest?	
7	Two rowers are canoeing in a river. They can row the canoe 24 miles downstream in 3 hours, but it takes them 9 hours to row 18 miles upstream. If the rowers always row at the same rate, what is the rate of the current of the river, in miles per hour?	
8	Five children (Andrew, Bob, Charlie, Diane, and Ellen) are to ride on the merry-go-round that has five seats around its outer edge. However, Diane and Ellen, being BFFs, demand that they sit in adjacent seats. How many distinguishable arrangements of the five children will accommodate Diane's and Ellen's demand? Any two seats are indistinguishable, so, for example, one arrangement of children is the same as the arrangement where each child moved one seat to his/her right.	
9	Amy traveled 200 meters on her bicycle to her friend's house. If the bicycle's wheels have a diameter of 60 centimeters, then to the nearest whole number, how many full rotations did the front wheel make?	
10	There are 128 bacteria in a culture at the beginning of an experiment. If the bacteria grow exponentially such that the culture size has increased by 50% in one hour, what will be the size of the culture after 7 hours?	

6 points each		
11	A math tournament has 20 participants. If one school sent three participants, what is the probability the school had one student place first, two students place in the top three, and all three students place in the top five? Assume no two students tied.	
12	Alliemarie is standing inside a large circular sand pit. If she walks directly north, it is 18 meters to the boundary of the pit. If she walks directly south, it is 24 meters to the boundary of the pit. If she walks directly east, it is 54 meters to the boundary of the pit. Find the area, in square meters, of the sand pit.	
13	John forgot how much money he loaned his mom, but two years later, she paid him back plus loan-shark interest (50%), compounded yearly. It turns out John only made $5 in interest on the deal. How much money, in dollars, did John loan his mom?	
14	Movie tickets to see the horrible new Teenage Mutant Ninja Turtles movie in IMAX 3D cost $15 per student and $19 per adult. If a group of people spent $638 to go see the horrible new Teenage Mutant Ninja Turtles movie in IMAX 3D, what is the largest number of people there could be in this group? (By the way, did I mention this movie is horrible?)	
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7	A giant, inverted, right, circular cone with radius 50 meters and height 25 meters is being filled with water at a rate of 1 cubic meter/second. How fast, in meters/second, is the depth of water in the cone changing when the water is 5 meters deep?	
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9	A farmer wants to enclose a rectangular region whose enclosed area is 3000 square feet. However, for ease, he wants at least two of the sides of the region to be positive integer numbers of feet. What is the least amount, in feet, of fencing necessary to do so?	
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7	Two rowers are canoeing in a river. They can row the canoe 24 miles downstream in 3 hours, but it takes them 9 hours to row 18 miles upstream. If the rowers always row at the same rate, what is the rate of the current of the river, in miles per hour?	3
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9	Amy traveled 200 meters on her bicycle to her friend's house. If the bicycle's wheels have a diameter of 60 centimeters, then to the nearest whole number, how many full rotations did the front wheel make?	106
10	There are 128 bacteria in a culture at the beginning of an experiment. If the bacteria grow exponentially such that the culture size has increased by 50% in one hour, what will be the size of the culture after 7 hours?	2187

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13	In how many ways can Yilan distribute 18 identical pieces of her birthday candy among herself, her nine classmates, and her teacher, assuming that she is going to be nice and give all eleven of us at least one piece?	19,448
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4	Two gears with radii of lengths 1 and 5 cm are positioned so that their centers are 8 cm apart. What is the minimum length of belt needed in order to fit completely around the two gears? The belt may not cross itself.	$\frac{22\pi + 24\sqrt{3}}{3}$ or equiv.
5	Ishant drops a ball from the top of a 250-foot tall building. The ball falls straight down, and each time it bounces off the ground, it rebounds to $\frac{1}{6}$ of the height from which it fell. What is the total distance traveled by the ball, in feet, before it comes to rest?	350

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Applications Solutions

Mu	Al	Th	Solution
1	1	1	Danae's experiment comes up 2.5 moles short of the maximum yield of 25 moles, so her percent error is 10%.
2	2	2	If x is the number of hours it takes to complete the house, then Connor paints $\frac{x}{5}$ of the house while Zheye paints $\frac{x}{8}$ of the house. Therefore, $\frac{x}{5}+\frac{x}{8}=1\Rightarrow\frac{13}{40}x=1\Rightarrow x=\frac{40}{13}$.
		3	Let t be the number of hours the first train takes to travel the common amount. Then $40t=50(t-1)=50t-50\Rightarrow 10t=50\Rightarrow t=5$. Since the first train left at 6 am and traveled for 5 hours, the time when the trains have traveled the same amount is 11 am.
3	3	4	Let x be the number of mL of the 50% acid solution that is needed. Then $0.5x+0\cdot 10=0.3(x+10)=0.3x+3\Rightarrow 0.2x=3\Rightarrow x=15$.
	4	5	Let s be the son's current age. This makes the daughter's current age $2s$ and the father's current age $8(2s)=16s$. Therefore, $38=s+2s+16s=19s\Rightarrow s=2$. Therefore, in ten years, the son will be 12 years old.
4			The length of the belt between the points of tangency on either side are $\sqrt{8^2-(5-1)^2}=4\sqrt{3}$, which means that the angle between the radius drawn to the point of tangency on the larger circle and the line between the centers is 60°, meaning that the belt accounts for $\frac{2}{3}$ of the larger gear's circumference and $\frac{1}{3}$ of the smaller gear's circumference. Therefore, the total length of belt is $\frac{2}{3}\cdot 2\pi\cdot 5+\frac{1}{3}\cdot 2\pi\cdot 1+2\left(4\sqrt{3}\right)=\frac{22\pi}{3}+8\sqrt{3}=\frac{22\pi+24\sqrt{3}}{3}$.
5	5	6	Because the distances both upward and downward form geometric sequences, the total distance the ball travels downward is $\frac{250}{1-\frac{1}{6}}=300$ and the total distance the ball travels upward is $\frac{\frac{250}{6}}{1-\frac{1}{6}}=50$. Therefore, the ball travels a total of 350 feet.
6	6		The area of the base is $\frac{3(4)^2\sqrt{3}}{2}=24\sqrt{3}$ and the height of the box is 4. Therefore, the volume of the box is $24\sqrt{3}\cdot 4=96\sqrt{3}$.
	7	7	The rowers average 8 miles per hour downstream and 2 miles per hour upstream. If r is the rate of the rowers and c is the rate of the current, then $r+c=8$ and $r-c=2$. Solving this system for c yields $c=3$.

7			Let r and h be the radius and height, respectively, of the water cone in the tank. We know that $r=2h$ since the water cone and the tank cone are similar. Therefore, $V=\frac{1}{3}\pi(2h)^2 h=\frac{4}{3}\pi h^3 \Rightarrow \frac{dV}{dt}=4\pi h^2\frac{dh}{dt} \Rightarrow 1=4\pi\cdot 5^2\frac{dh}{dt} \Rightarrow \frac{dh}{dt}=\frac{1}{100\pi}$.
8	8	8	Since the seats are indistinguishable, there is really only one way for Diane to be seated. Once Diane is seated, Ellen then has two possible seats. The other three children can be positioned in any way, which is $3!=6$ ways, so there are a total of $1\cdot 2\cdot 3!=12$ ways for Diane and Ellen to be seated together. OR There are five possible seats for Diane, and once Diane is seated, Ellen then has two possible seats. The other three children can be positioned in any way, which is $3!=6$ ways. However, since Diane's seat placement relative to the other children is the same no matter which seat is Diane's, there are a total of $\frac{5\cdot 2\cdot 3!}{5}=12$ ways for Diane and Ellen to be seated together.
		9	One rotation of the tires is 60π centimeters, and the total trip is 20,000 centimeters, so the total number of rotations of the tires is $\frac{20{,}000}{60\pi}\approx 106.1$, so to the nearest whole number, the wheels made 106 rotations.
	9	10	Based on the given conditions, the growth model for this bacteria culture is $P=128\left(\frac{3}{2}\right)^t$, where t is given in hours. Therefore, after 7 hours, the culture population is $P=128\left(\frac{3}{2}\right)^7=3^7=2187$.
9			It is fairly easy to show that since fencing must be on all four sides that the minimum amount of fence needed is when the region is a square, making the side lengths for this configuration $\sqrt{3000}\approx 54.7$, but this will not satisfy the positive integer length condition. Therefore, trying one dimension to be 54 or 55 feet, the other dimension must be $\frac{500}{9}=55\frac{5}{9}$ or $\frac{600}{11}=54\frac{6}{11}$ feet, respectively. Since $\frac{6}{11}<\frac{5}{9}$, the minimum amount of fencing needed is $2\left(55+\frac{600}{11}\right)=2\left(\frac{1205}{11}\right)=\frac{2410}{11}$ feet.
10	10		The total area enclosed by the circle is $\pi\cdot 4^2=16\pi$, and the maximum number of regions created in the interior of the circle by 12 lines is $\frac{12\cdot 13}{2}+1=79$. Therefore, the average area of the pieces is $\frac{16\pi}{79}$.

11	11	11	The placement of students essentially means one student won, one student was second or third, and the third student was fourth or fifth. We only need order the top five spots. There are $\binom{3}{1}=3$ ways to pick the first place student. For the next two spots, there are $\binom{2}{1}=2$ ways to pick the student from the school, $\binom{17}{1}=17$ ways to pick a student from the other schools for the remaining place, and $\binom{2}{1}=2$ ways to order those two students. For the final two spots, there is $\binom{1}{1}=1$ way to pick the student from the school, $\binom{16}{1}=16$ ways to pick a student from the others schools for the remaining place, and $\binom{2}{1}=2$ ways to order those two students. For those five spots, there are ${}_{20}P_5$ possible orderings, so the probability is $\frac{3\cdot2\cdot17\cdot2\cdot1\cdot16\cdot2}{20\cdot19\cdot18\cdot17\cdot16}=\frac{1}{285}$.
12	12	12	Let *x* be the distance Alliemarie must walk west to reach the boundary of the pit. Drawing the segments from her position to the boundary of the pit creates two chords. Because the two chords intersect, $x\cdot54=18\cdot24\Rightarrow x=8$. Reflect the chord of total length 62 across the diameter parallel to it, so the distance between this chord and its reflection is 6, making the distance from the center of the circle to one of those chords 3. Further, half of the reflected chord has length 31, so the radius of the circle is $r=\sqrt{3^2+31^2}=\sqrt{970}$, making the enclosed area 970π.
		13	$P+5=P\left(1+\frac{0.5}{1}\right)^{1\cdot2}=2.25P\Rightarrow1.25P=5\Rightarrow P=4$
13	13		Have Yilan go ahead and distribute one piece to all eleven people. That leaves seven pieces of candy to distribute, including giving some people 0 additional pieces. The total number of ways to distribute 7 identical pieces of candy among 11 people accounting for the possibility of some people receiving none is $\binom{7+11-1}{7}=\binom{17}{7}$ $=19,448$.
	14	14	Let *s* and *a* be the number of students and adults, respectively, who attend this horrible movie. Then $15s+19a=638$, where *s* and *a* are whole numbers. One solution that is easy to spot is $s=40$ and $a=2$. To find all other solutions in integers, because 15 and 19 are relatively prime, add or subtract 19 from *s* while subtracting or adding 15, respectively from/to *a*. Therefore, increasing *s* while decreasing *a*, the next solution is $s=59$ and $a=-13$, which is not a valid solution to this problem. Therefore, we will only subtract from *s* until it becomes a negative integer (and add to *a*). Other possible solutions are $s=21$ and $a=17$, and $s=2$ and $a=32$ (the next solution would be $s=-17$ and $a=47$, which does not work). Of the three possible solutions, in the order they are listed here, the total number of people attending the horrible movie is 42, 38, and 34. Therefore, the maximum number of people attending is 42.

14	15	15	Nine plates could be traced in three rows, tangent to all edges, so a sheet not longer than 36 inches would suffice. However, the most efficient packing of circular regions in a plane is the one where any three adjacent circles are all externally tangent, creating an equilateral triangle with the three centers of those adjacent circles. Therefore, the shortest length would be three rows of 3, 2, and 3 circles. The length would need to accommodate 2 radii and 2 altitudes of the aforementioned equilateral triangles. Since the sides of the equilateral triangles each have length 12 inches, the minimum length is $2(6)+2\left(6\sqrt{3}\right)=12+12\sqrt{3}$.
15			If one wall has length *x*, the other wall has length $400-x$. Let θ be the angle between the two walls. Therefore, the area of the region is $\frac{1}{2}x(400-x)\sin\theta$. Since $0^\circ<\theta<180^\circ$, all other things being equal, a value of $\theta=90^\circ$ would give a greatest sine value (1). To maximize $\frac{1}{2}x(400-x)$, recognize that $y=\frac{1}{2}x(400-x)$ is a parabola opening downward with *x*-intercepts of 0 and 400, so its vertex must occur when $x=200$ and is a maximum. Therefore, the maximum area occurs when $x=200$ and $\theta=90^\circ$, making the enclosed area $\frac{1}{2}(200)(400-200)(1)=20{,}000$.